Chapter 2 - Sets

Definitions

- A **set** is a collection of objects

- An **element/member** is an object in a set

Examples of sets

- The collection of whole numbers
- The collection of people who have been in a movie with Kevin Bacon
- The collection of people who wrote a whole book on a Thursday, speak multiple languages, and have a last name starting with ‘A’.

There are two common ways to list sets:

- Roster Notation
  lists every element in the set

- Set Builder
  provides a rule to find all elements in a set

Both notations are placed within braces

{ }

Roster Notation (lists every element)

Examples:

{ 1, 2, 3, 4, 5 }

{ Cincinnati, Cleveland, Columbus }

Ellipsis "…" can be used to show that elements continue in the same manner.

Examples:

{ 1, 2, 3, 4, 5, … , 10 }

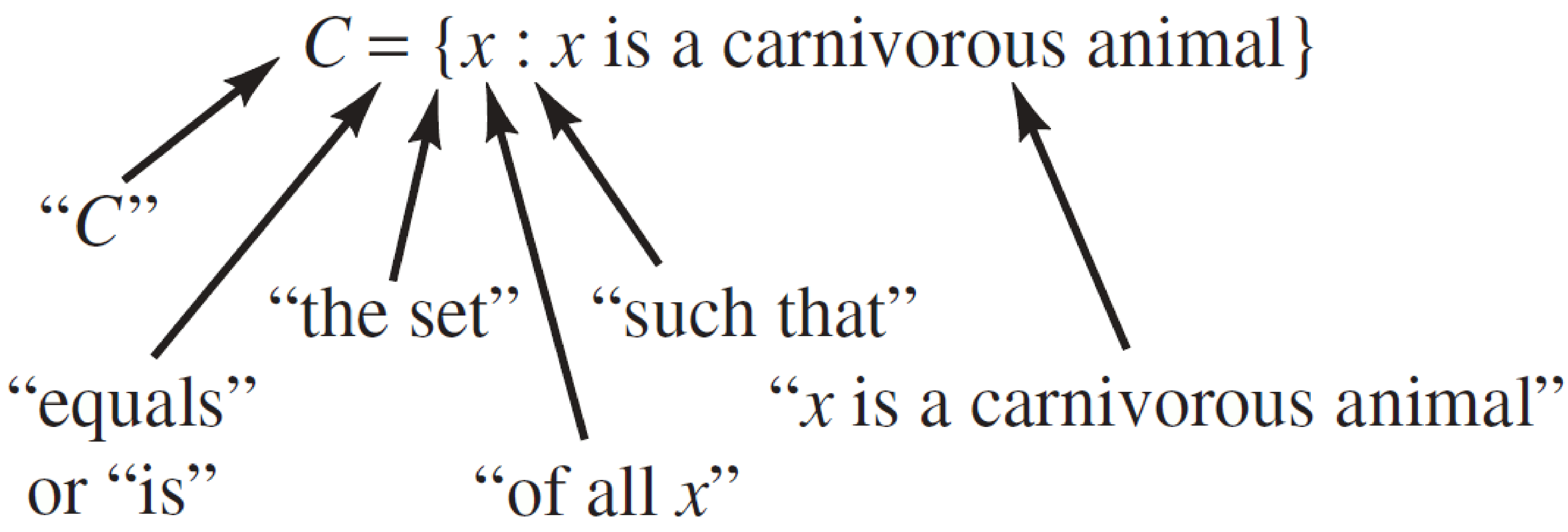
{ 0, 1, 2, 3, 4, 5, 6, … }

{ … , -3, -2, -1, 0, 1, 2, 3, … }

{ Washington, Adams, … , Bush, Obama }

Representing Sets

- Set-builder notation:



Examples of sets in Set Builder Notation

A = { x : x is a whole number and
1 <= x and x <= 10 }

B = { x : x was in a movie with Kevin Bacon }

C = { x : x is a complete graph }

Representing Sets

- A set is *well-defined* if we are able to tell whether any particular object is an element of the set.

- Example: Which sets are well-defined?

$$A = \{\, x : x \text{ is a winner of an Academy Award} \}$$

$$T = \{\, x : x \text{ is tall} \}$$

How do we represent the following set in Roster Form?

{ x : x is a student at UT and
x is a space alien }

{ }

A set with no entries is known as the empty set. It can also be written as

$\varnothing$

Representing Sets

DEFINITION The set that contains no elements is called the **empty set** or **null set.** This set is labeled by the symbol ∅. Another notation for the empty set is { }.

- Do ∅ and {∅} mean the same thing?
  - ∅ is the empty set – a set with no members
  - {∅} is a set with a member object, namely, the empty set

The previous example shows that is possible to have a set of sets. (And sets of sets of sets, and …)

Examples:

{ 1, { 1 } }

{ { 1, 2, 3, 4, 5, … } , {-1, -2, -3, -4, -5, … } }

{ { { 3, 5 }, { 1 } } , { Kevin Bacon } }

The Element Symbol

$\in$ means "is an element of"
$\notin$ means "is *not* an element of"

- Example:

$$3 \in \{2,3,4,5\}$$

$$6 \notin \{2,3,4,5\}$$

K_4 { x : x is a complete graph }

Representing Sets

DEFINITION The **universal set** is the set of all elements under consideration in a given discussion. We often denote the universal set by the capital letter *U*.

- Example: Consider female consumers living in the U.S. The universal set is

$$U = \{\, x : x \text{ is a female cosumer living in the U.S.} \}$$

Cardinal Number

DEFINITIONS The number of elements in set *A* is called the **cardinal number** of set *A* and is denoted $n(A)$. A set is **finite** if its cardinal number is a whole number. An **infinite** set is one that is not finite.

$$n(A)$$

The *n* reminds us of the word "number."

Capital *A* reminds us that we are dealing with a set.

- Example: State the cardinal number of the set.

$$X = \{\{1,2,3\},\{1,4,5\},\{3\}\}$$

$$n(X) = 3$$

(the set X contains 3 objects, each of which is also a set)

Set Equality and Subsets

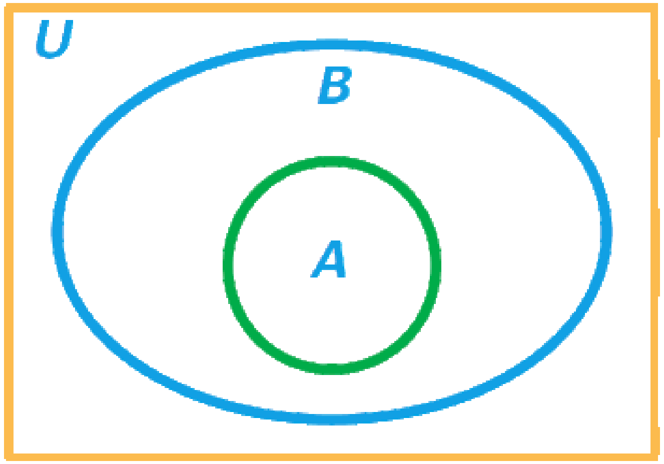
DEFINITION Two sets A and B are **equal** if they have exactly the same members. In this case, we write $A = B$. If A and B are not equal, we write $A \neq B$.

DEFINITION The set A is a **subset** of the set B if every element of A is also an element of B. We indicate this relationship by writing $A \subseteq B$. If A is not a subset of B, then we write $A \not\subseteq B$.

Venn Diagrams and Proper Subsets

- A *Venn diagram* is used to visualize relationships among sets.

DEFINITION The set A is a **proper subset** of the set B if $A \subseteq B$ but $A \neq B$. We write this as $A \subset B$. If A is not a proper subset of B, then we write $A \not\subset B$.



The Venn Diagram of the different types of numbers

Venn Diagrams and Proper Subsets

THE NUMBER OF SUBSETS OF A SET A set that has *k* elements has 2^k subsets.

- How many subsets exist for the given set?

$$A = \{\text{Bill, Gill, Jill, Will}\}$$

$$2^k = 2^4 \quad 16$$

Equivalent Sets

DEFINITION Sets *A* and *B* are **equivalent,** or in **one-to-one correspondence,** if $n(A) = n(B)$. Another way of saying this is that two sets are equivalent if they have the same number of elements.*

- The sets {1, 2, 3} and {A, B, C} are equivalent because they both have 3 members.